

Langlands duality for Hitchin systems

Abstract

In this talk we will state and outline a proof of the classical limit of the geometric Langlands conjecture, and discuss its relation to the full "quantum" conjecture. Concretely, we show that the Hitchin integrable system for a simple complex Lie group G is dual to the Hitchin system for the Langlands dual group ${}^L G$. In particular, the general fiber of the connected component \mathbf{Higgs}_0 of the Hitchin system for G is an abelian variety which is dual to the corresponding fiber of the connected component of the Hitchin system for ${}^L G$. The non-neutral connected components \mathbf{Higgs}_α form torsors over \mathbf{Higgs}_0 . We show that their duals are gerbes over \mathbf{Higgs}_0 which are induced by the gerbe of G -Higgs bundles \mathcal{Higgs} . More generally, we establish a duality between the gerbe \mathcal{Higgs} of G -Higgs bundles and the gerbe ${}^L \mathcal{Higgs}$ of ${}^L G$ -Higgs bundles, which incorporates all the previous dualities. All these results extend immediately to an arbitrary connected complex reductive group \mathbb{G} .